

# Block-Cipher Cascading Strikes Back: Tight Bounds for Security Amplification

Stefano Tessaro

ETH Zurich

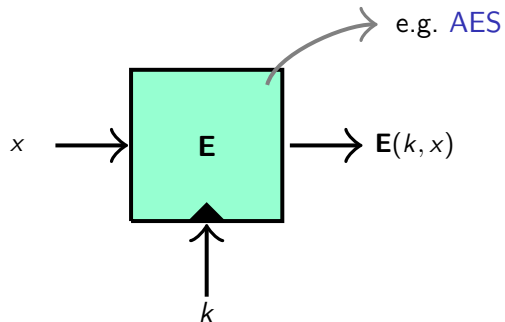
Rump Session EUROCRYPT 2010



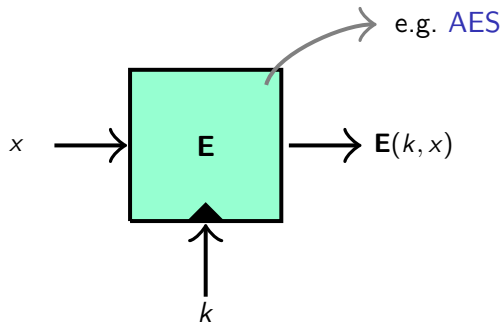
# Motivation: Block Ciphers

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## Block Cipher



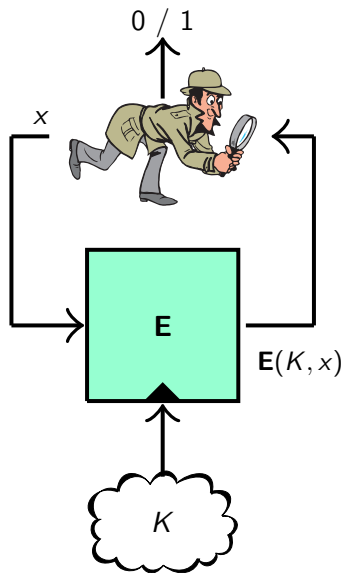
## Block Cipher



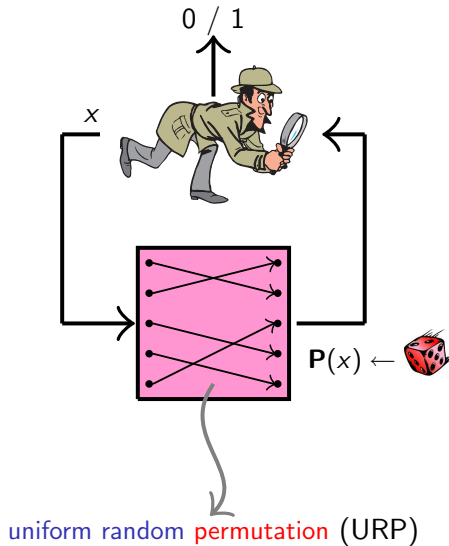
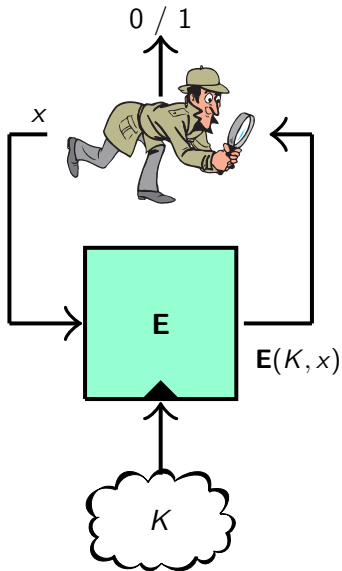
Security notion: **Pseudorandom Permutation**

## Motivation: Block Ciphers – Pseudorandom Permutations

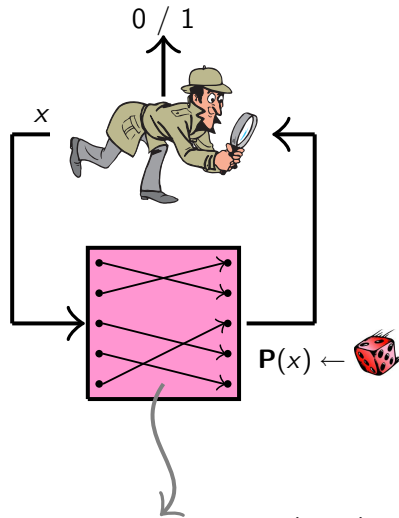
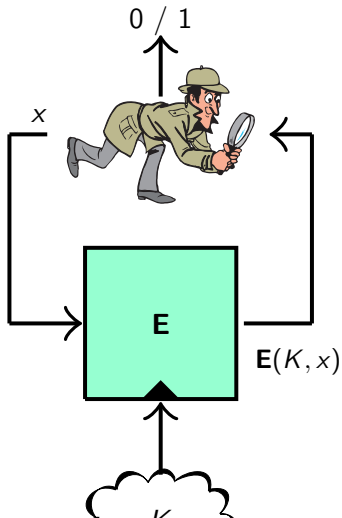
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# Motivation: Block Ciphers – Pseudorandom Permutations

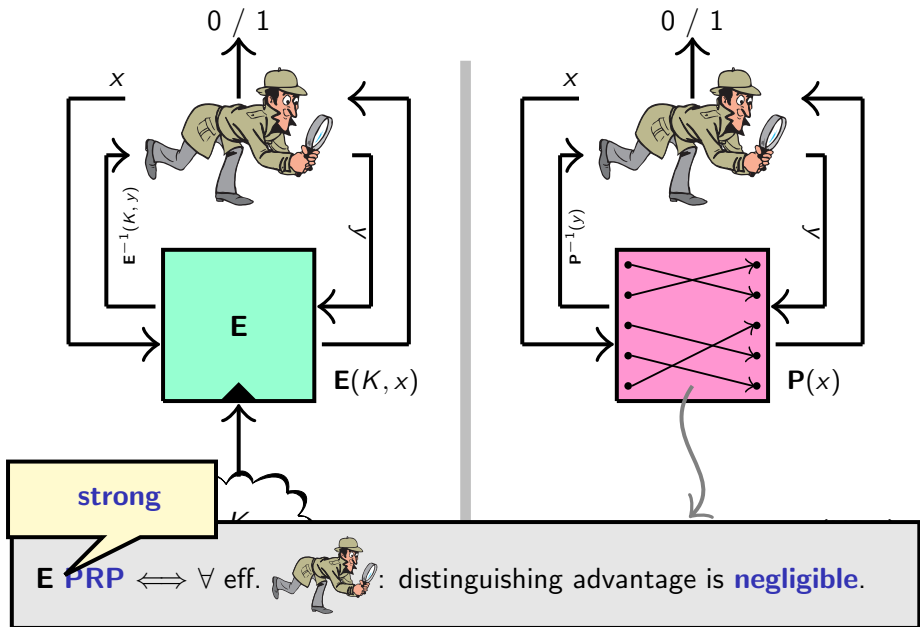


# Motivation: Block Ciphers – Pseudorandom Permutations

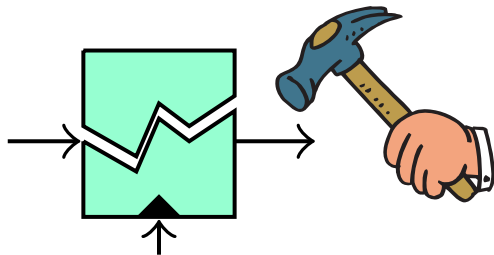


$E$  PRP  $\iff \forall$  eff.  : distinguishing advantage is **negligible**.

# Motivation: Block Ciphers – Strong Pseudorandom Permutations

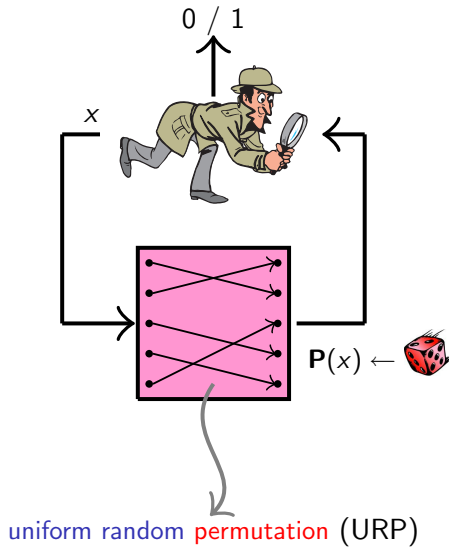
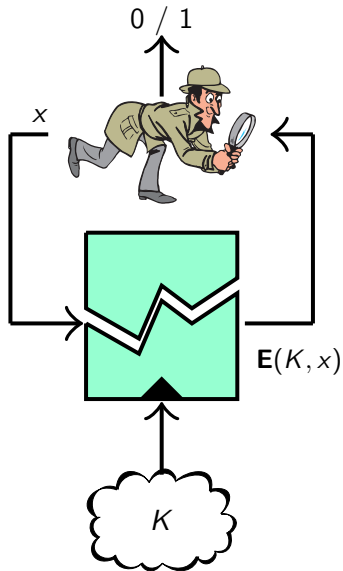


## Block Ciphers get broken!

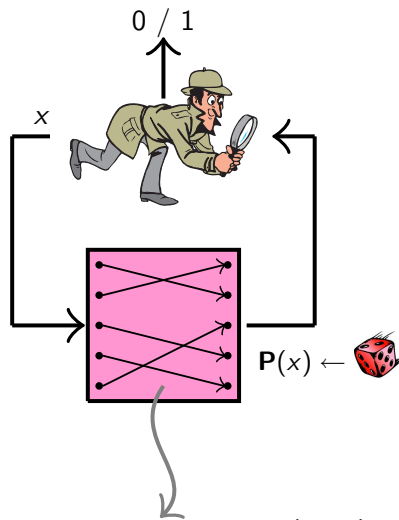
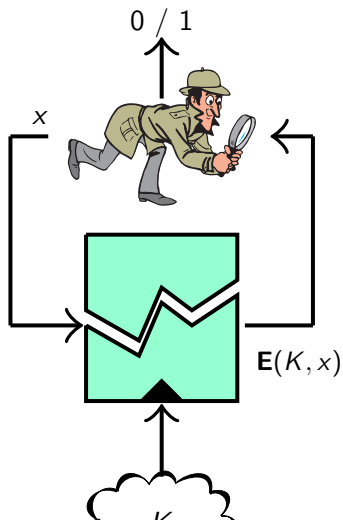





# Weak Block Ciphers

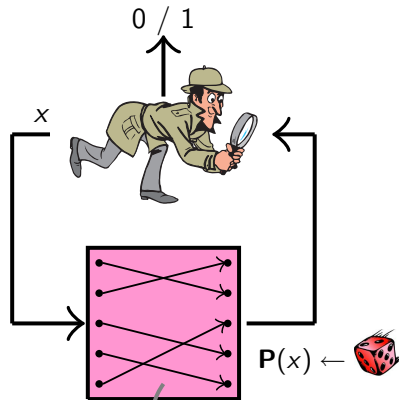
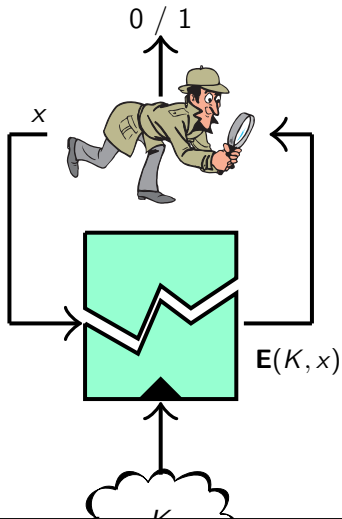


# Weak Block Ciphers



$E$   $\epsilon$ -PRP  $\iff \forall$  eff.  : distinguishing advantage is  $\leq \epsilon$ .

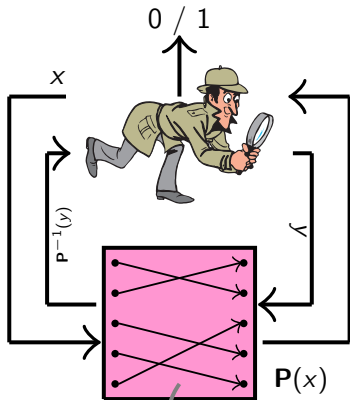
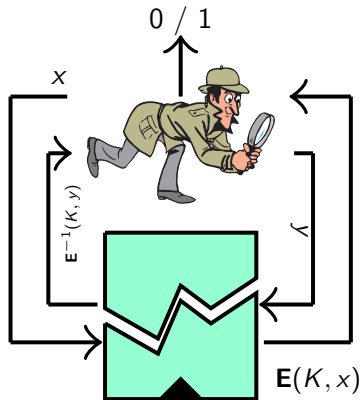
# Weak Block Ciphers



$$\epsilon = \frac{1}{k}, 0.5, 0.99, 1 - \frac{1}{k}, \dots$$


$E$   $\epsilon$ -PRP  $\iff \forall$  eff.  distinguishing advantage is  $\leq \epsilon$ .

# Weak Block Ciphers

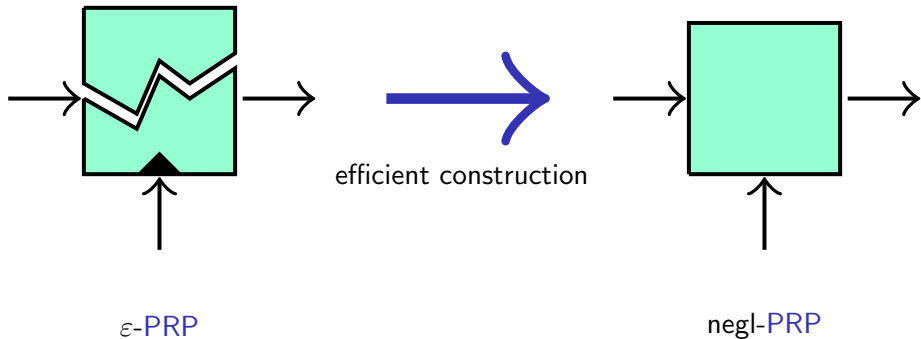


strong

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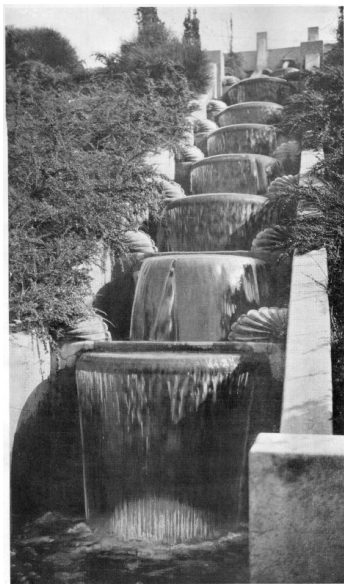
$E$   $\epsilon$ -PRP  $\iff \forall$  eff.  : distinguishing advantage is  $\leq \epsilon$ .

# Security Amplification



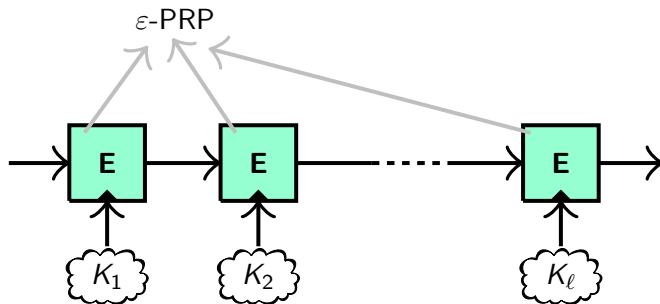
# Cascades of Block Ciphers

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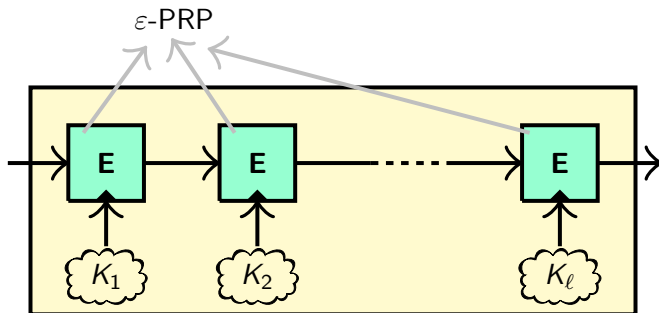
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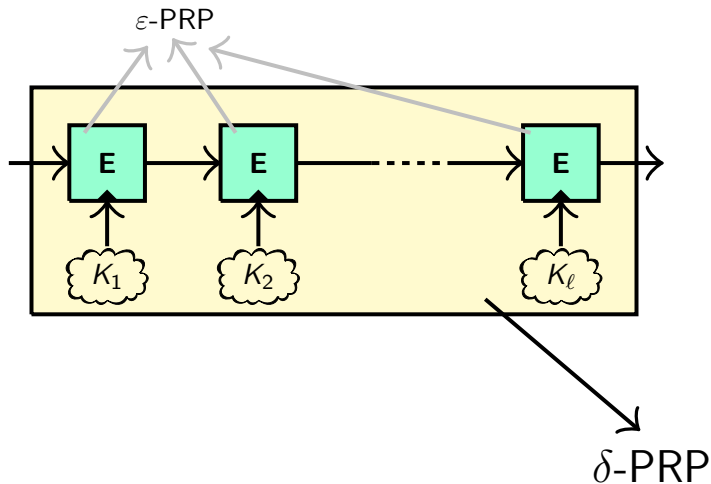
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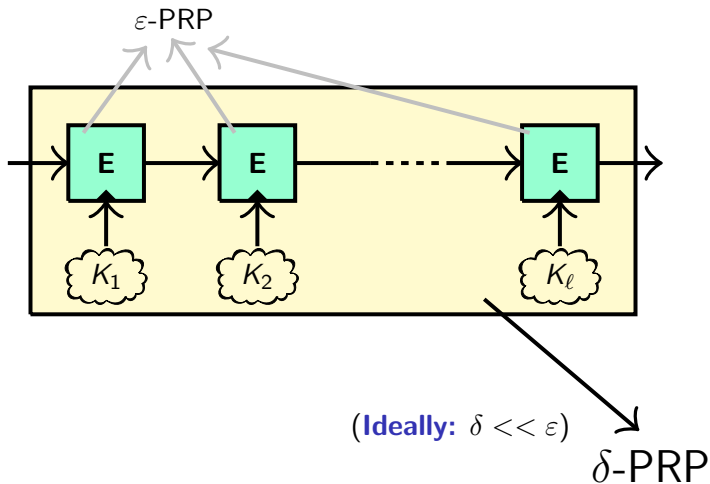




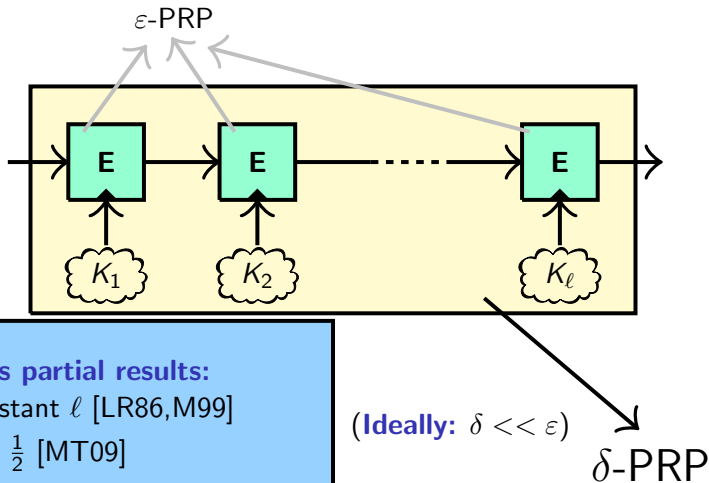
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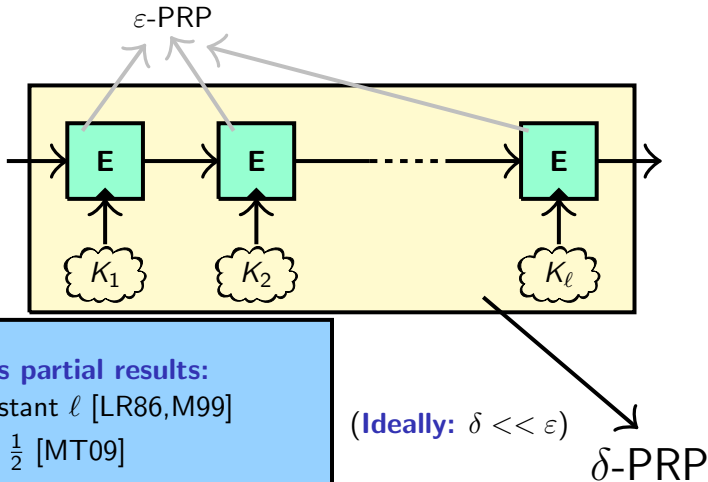
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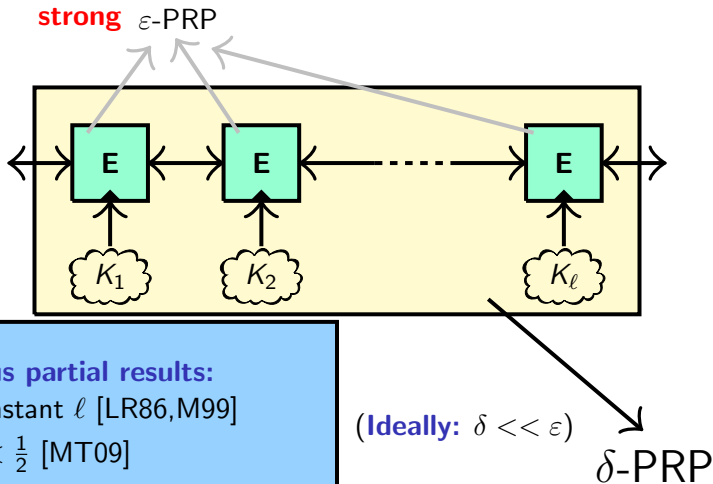
# Cascades of Block Ciphers



**Our new bound:**  $l$ -cascade is  $(\epsilon^l(l - (l - 1)\epsilon) + \text{negl})$ -PRP



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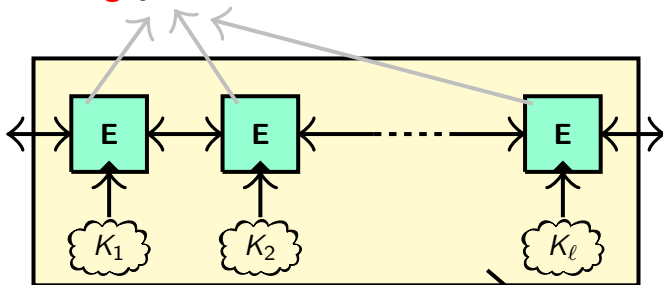
Previous partial results:

- ▶ constant  $l$  [LR86, M99]
- ▶  $\epsilon < \frac{1}{2}$  [MT09]

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strong

strong  $\epsilon$ -PRP



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(Ideally:  $\delta \ll \epsilon$ )

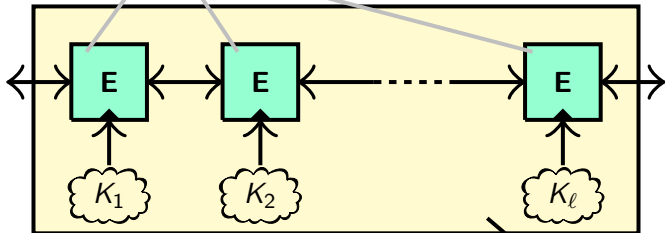
$\delta$ -PRP

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Security amplification  $\forall \epsilon < 1 - \frac{1}{|\text{Domain}|}$

strong

strong  $\epsilon$ -PRP



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## Final Remarks

- ▶ Bounds are **tight**
- ▶ **New technique** based on **interactive Hardcore Lemma**



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## Paper

“Security Amplification for the Cascade of Arbitrarily Weak PRPs:  
Tight Bounds via the Interactive Hardcore Lemma”

[www.crypto.ethz.ch/publications/](http://www.crypto.ethz.ch/publications/)