Extremal Pruning: Applying Game-Hopping Techniques in Real Life

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Game-Hopping Proofs

- Game-hopping proofs are an incredibly useful tool in mathematical cryptography.
- Key point: Memoryless games.

- Can this approach be used to analyse real-life situations?

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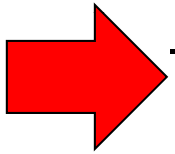

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$$\Pr[W_1] = 1$$

➢ Help me!

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$$\Pr[W_2] = \tfrac{1}{2}\Pr[W_1]$$

- They've trapped me in this bizarre operating theatre.

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- In Game 2, the subject had “lost” one randomly chosen arm.
- Game 3 is identical to Game 2 except we hack off both of the subjects arms with a rusty machete (or dirty scythe).

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$$\Pr[W_3] = \tfrac{1}{2}\Pr[W_2]$$

- It’s very dark and I’m scared. I found this laptop on a table.

Conclusion

$$\Pr[W_3] = \tfrac{1}{4} \Pr[W_1] = \tfrac{1}{4}$$

- Even with no arms, you can pick up a ball on a table about one time in four.

➢ There is some rusty farm equipment in the corner.

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- How practical is our (rather literal) reduction?
- How tight are our bounds?
  - Our bounds are very tight...

➢ I think I hear someone coming... and they're laughing.

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- What about our assumptions?
- Experimental evidence suggests that subjects don't pick up the ball after their favoured hands have been removed (with a rusty machete, dirty scythe, or soiled scalpel).
  - Mostly they just scream and bleed.

➢ Oh God! No! NO!! Tell my wife I love her...