What Price a Provably Secure Cipher?

Bo-Yin Yang

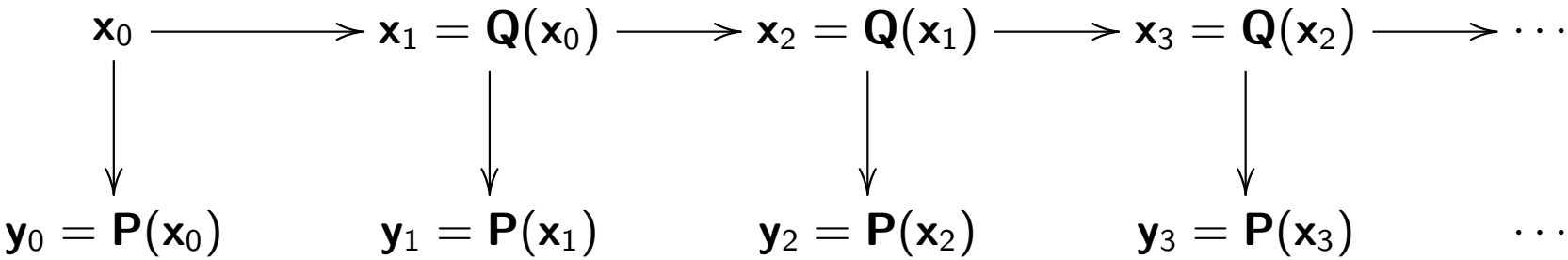
Institute of Information Sciences
Academia Sinica
Taipei, Taiwan
byyang@iis.sinica.edu.tw

June 1, 2010, Eurocrypt

The Provably-secure QUAD(q, n, r) Stream Cipher

- Proposed by Berbain, Gilbert, and Patarin in Eurocrypt 2006
- P_i's, Q_j's: randomly chosen, public quadratic polynomials

State: n-tuple $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in \mathbb{F}_q^n$
Output: r-tuple $(P_1(\mathbf{x}), P_2(\mathbf{x}), \ldots, P_r(\mathbf{x}))$
Update: $\mathbf{x} \leftarrow (Q_1(\mathbf{x}), Q_2(\mathbf{x}), \ldots, Q_n(\mathbf{x}))$



Security of QUAD

- Main security theorem of QUAD
  - Breaking QUAD implies the capability to solve $n + r$ random quadratic equations in n variables
- Generic $\mathcal{MQ}$ (Multivariate Quadratics) is NP-hard
  - $\mathcal{MQ}(q, n, n + r) =$ solve for n variables from $n + r$ quadratic equations, all coefficients and variables in $\mathbb{F}_q$
  - All known algorithms have average time complexity $2^{an+o(n)}$ for $r/n =$ constant
    - Most also require exponential space

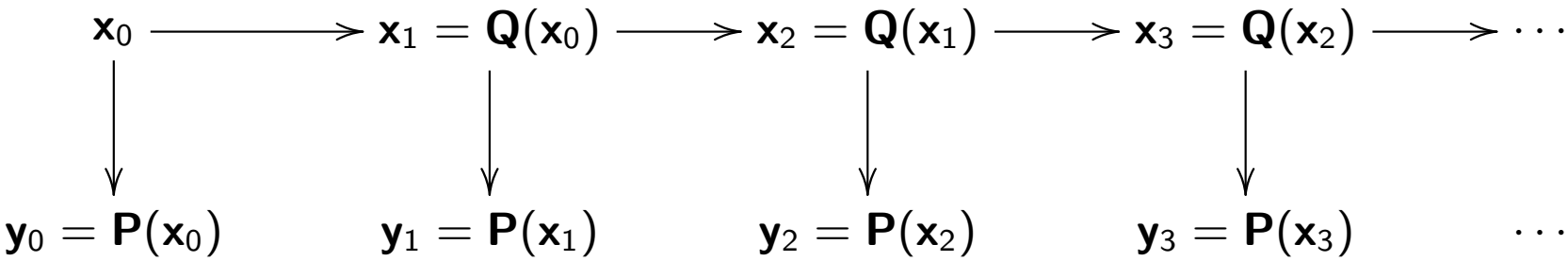
Key Observation

- The same reduction carries over to polynomials of *arbitrary* degrees, e.g., cubics, quartics, . . . , *without any modifications*
  - So long as linear terms are dense to keep the same distribution under random linear forms
  - But polynomials with higher degrees have way too many coefficients to be practical!
    - Need to use sparse polynomials
    - Need a new security assumption

$\mathcal{SMP}(q, d, n, m, (\eta_2, \ldots, \eta_d))$

- An instance **S** in $\mathcal{SMP}(q, d, n, m, (\eta_2, \ldots, \eta_d))$, the class of *sparse multivariate polynomials*, comprises
  - m polynomials $(P_1(\mathbf{x}), P_2(\mathbf{x}), \cdots, P_m(\mathbf{x}))$ in n variables $\mathbf{x} = (x_1, x_2, \ldots, x_n)$
  - Each P_i is a degree-d polynomial with exactly $\eta_j = \eta_j(n)$ nonzero degree-j terms for each $2 \leq j \leq d$
  - The affine terms are random
- Obviously $\mathcal{SMP}$ contains $\mathcal{MQ}$
- Furthermore, solving $\mathcal{SMP}$ systems with reasonably many terms appears to be hard
  - Ample empirical evidence to support this conjecture

SPELT, Generalization of QUAD



1. $\mathbf{P}$,$\mathbf{Q}$ drawn from $\mathcal{SMP}$
2. Need to select good parameters, say for $q = 16, n = r$
   - For cubics, need $n = 144$ at least
   - For quartics, need $n = 108$ at least
   - Don't need too many terms
     - 10 cubic terms per equation already makes things hard

Timing on 3 GHz Intel CPU

Stream cipher	Cycles/byte	Throughput	Security
AES (Bernstein and Schwabe)	9.2	2.61 Gbps	$\leq 2^{?}$
SPELT$(16, 4, 32, 32, (10, 8, 5))$	1244	19.3 Mbps	$\leq 2^{152}$
QUAD$(2, 160, 160)$ (BBG SAC 2006)	2081	11.5 Mbps	$\leq 2^{140}$
SPELT$(16, 4, 108, 108, (20, 15, 10))$	5541	4.3 Mbps	$\geq 2^{80}$
SPELT$(2, 3, 208, 208, (480, 20))$	11744	2.0 Mbps	$\geq 2^{82}$
QUAD$(2, 320, 320)$ (BBG SAC 2006)	13646	1.8 Mbps	$\geq 2^{82}$

Latest Development

- We learned how to launch better brute-force attacks
  - $O(2^n)$ rather than $O(2^{n+o(n)})$
  - Bad news for QUAD/SPELT because this means more variables and slower speed
- We learned how to program GPU
  - Can we make QUAD/SPELT usable in practice?

Preliminary Performance Results

Stream cipher	Cycles/byte	Throughput	
		C2Q 9550	GPU
AES (BS; OBSC, FSE 2010)	9.2	2.61 Gbps	30.9 Gbps
SPELT(64, 4, 32, 32, (10, 8, 5))	1244	19.3 Mbps	
QUAD(2, 160, 160) (BBG SAC 2006)	2081	11.5 Mbps	
SPELT(16, 4, 108, 108, (20, 15, 10))	5541	4.3 Mbps	
SPELT(2, 3, 208, 208, (480, 20))	11744	2.0 Mbps	
QUAD(2, 320, 320) (BBG SAC 2006)	13646	1.8 Mbps	
SPELT(31, 4, 96, 96, (32, 16, 8))	549	43.7 Mbps	914 Mbps
SPELT(16, 4, 96, 96, (32, 16, 8))	573	41.9 Mbps	784 Mbps
SPELT(2, 3, 224, 224, (448, 20))	3121	7.3 Mbps	826 Mbps
QUAD(2, 320, 320)	3701	6.1 Mbps	2.6 Mbps

Concluding Remarks

- In the case of stream cipher, the cheapest price for provable security seems to be one or two orders of magnitude in terms of speed

Acknowledgement

- This is a joint work with
  - Tien-Ren Chen
  - Chun-Hung Hsiao
  - Ruben Niederhagen
  - Ming-Shing Chen
  - Chen-Mou Cheng, National Taiwan University